Intermission Chapter – For Review and Self-study

- Up to here, we introduced most of QM's structure
- Before moving on, sit down and think through...
  - Wavefunction $\Psi(x,t)$ [contains all information of a state]
  - Interpretation $|\Psi(x,t)|^2 dx$
  - Well behaved Ψ
  - TDSE is the governing wave equation, it governs how Ψ evolves in time
  - TDSE includes information on dimensionality (1D, 2D, 3D) and what the system is about ($U(x,t)$ or $U(x)$)

- For $U = U(x)$ only, TDSE separates into TISE + temporal equation

- State of definite energy E evolves as $\psi_E(x)\, e^{-iEt/\hbar}$

- Possible (allowed) energies E and states $\psi_E(x)$ are solutions to TISE

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + U(x)\psi(x) = E\psi(x)$$

$$\psi_1(x) \leftrightarrow E_1 \;;\; \psi_2(x) \leftrightarrow E_2 \;;\; \cdots \;;\; \psi_n(x) \leftrightarrow E_n \;;\; \cdots$$

TISE is to solve for many $\psi_i(x)$ and the corresponding E_i

- TISE is an eigenvalue problem of the operator $\hat{H}$

- Hamiltonian operator $\hat{H}$ can be constructed for a problem by:

$$H = k.e. + p.e \quad \text{(Think classical)}$$

$$x \to \hat{x} \to x \;;\; p \to \hat{p} \to \frac{\hbar}{i}\frac{d}{dx} \quad \text{(Go Quantum)}$$

- $\hat{H}$ has its root in classical Mechanics (Newton, Lagrange, Hamilton)

- TISE $\quad \hat{H}\psi = E\psi \qquad$ Eigenfunctions: States of definite energy

Eigenvalues: Allowed energies

- In terms of $\hat{H}$, TDSE is $\hat{H}\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$

- $\hat{x} \to x$ and $\hat{p} \to \frac{\hbar}{i}\frac{d}{dx}$ (1D cases) are the only operators needed to construct operators for other physical quantities

  $\hat{x}$ (position operator), $\hat{p}$ (momentum operator)

  $$[\hat{x}, \hat{p}] = i\hbar$$

- Any quantity in classical mechanics has a corresponding QM operator

  [2-step: Think Classical + Go Quantum]

- Operators in QM are linear operators [actually more than this, see later]

- A physical quantity A thus has an operator $\hat{A}$

$$\hat{A}\,\phi(x) = a\,\phi(x)$$ is the eigenvalue problem of $\hat{A}$

- result of a measurement of the quantity A must be an eigenvalue of $\hat{A}$, i.e.

$$\{a_1, a_2, \cdots, a_n, \cdots\}$$

one eigenvalue will show up as result

- Immediately after a measurement (say result is a_i), the wavefunction of the system becomes $\phi_i(x)$, the eigenfunction of $\hat{A}$ with eigenvalue a_i (the measured result)

- Hence, _if_ state before measuring A is an eigenfunction $\phi_j(x)$ of $\hat{A}$, the measurement outcome is 100% certain to be the eigenvalue a_j

- For general states before measurement, no such definitive (certainty) statement can be made. [We can make probabistic statement, see later]

The statements are the key concepts of QM. Some are postulates, some are mathematical consequences, and some are physical requirements.

The point is that QM, based on these concepts, has passed every test in nearly 100 years, has led to deeper understanding in atoms, molecules, solids, nuclei, and elementary particles. It also opened up new subjects and new industries.

Carry these concepts with you into the next part, where we will do QM calculations.